

## Suppression and enhancement of soliton switching during interaction in periodically twisted birefringent fibers

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Soliton interaction in periodically twisted birefringent optical fibers has been analyzed analytically with reference to soliton switching. For this purpose we construct the exact general two-soliton solution of the associated coupled system and investigate its asymptotic behavior. Using the results of our analytical approach we point out that the interaction can be used as a switch to suppress or to enhance soliton switching dynamics, if one injects a multisoliton as an input pulse in the periodically twisted birefringent fiber.

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It is a well-known fact that single-core fiber supports two distinct modes of propagation as a result of the birefringence effect, which can be introduced through twisting the fiber during the preform stage of formation or through the stress induced birefringence mechanism. For many years the propagation of solitons in stress induced birefringent nonlinear Kerr media with reference to optical fibers has been the subject of intensive research for theoretical as well as experimental investigations [1]. The topic of propagation in twisted birefringent optical fiber has also gained considerable interest theoretically and experimentally recently as most of the nonlinear directional couplers are based on such fibers. One may mention the example of the rocking rotator [1] which can be used as a switching device at high power [2] and as a filter at low power [3].

Soliton propagation in the periodically twisted birefringent fiber is usually described by using the coupled nonlinear Schrödinger (CNLS) family of equations [1,2]. However, such CNLS equations are in general not completely integrable. Interestingly, if we assume the value of the ellipticity angle to be  $35^\circ$  then the dynamics of soliton interaction in a periodically twisted birefringent fiber coupler can be described by coupled wave equations of the form [4]

$$iq_{1z} + q_{1tt} + \rho q_1 + \kappa q_2 + 2\mu(|q_1|^2 + |q_2|^2)q_1 = 0, \quad (1)$$

$$iq_{2z} + q_{2tt} - \rho q_2 + \kappa q_1 + 2\mu(|q_1|^2 + |q_2|^2)q_2 = 0,$$

where  $q_1(z,t)$  and  $q_2(z,t)$  are slowly varying envelopes of two orthogonally polarized modes,  $z$  and  $t$  are, respectively, the normalized distance and time, and  $\kappa$  and  $\rho$  are the normalized linear coupling constants caused by the periodic twist of the birefringence axes and the phase-velocity mismatch from resonance, respectively. If the linear coupling constants are absent (that is,  $\kappa = \rho = 0$ ), then one can easily recognize the system (1) to be the celebrated integrable Manakov model [5]. The resulting Manakov equation is receiving renewed attention recently as it describes the effects of averaged random birefringence on an orthogonally polarized pulse in a real fiber [6]. When the birefringence axes of a fiber are periodically twisted during the drawing process there is a periodic intensity exchange between the orthogonally polarized modes [1-4] and it can be modeled by Eq.

(1) if the value of the ellipticity angle is  $35^\circ$  [4]. The linear coupling length where the maximum power is transferred from one mode to the other is  $\pi/(2\sqrt{\kappa^2 + \rho^2})$ . At the resonance wavelength, the linear parameter  $\rho = 0$  and the linear coupling length increases to  $\pi/(2\kappa)$ . The schematic of the experimental apparatus used to observe switching characters in the periodically twisted birefringent fiber is given in Ref. [2]. Particularly the dependence of switching characters on the input power, operating wavelength, twist magnitude, and twist period are described, for example, in Refs. [2,4].

Considerable attention has been paid in the literature [1,5,7,8] to studying soliton collision in the birefringent fiber. Particularly by using the system (1) in the absence of linear coupling terms Manakov [5] pointed out that during soliton collision their velocities and amplitudes (intensities) do not change but the associated unit polarization vectors do change provided they are neither parallel nor orthogonal (see also the paper of Silmon-Clyde and Elgin [8] for a discussion in terms of Stokes vectors). Further, Menyuk [7] has observed at the value of the ellipticity angle of  $35^\circ$ , where the Manakov equation holds good, that a soliton of one polarization when interacting with a switching pulse of the other polarization does not develop a shadow and also does not change shape. However, very recently we have proved [9] by constructing the most general two-soliton solution of the Manakov model that it has the property that a soliton in a birefringent fiber can in general change its shape after interaction due to a change in intensity distribution among the modes even though the total energy is conserved. In this paper we investigate the implication of this property of the solitons when the additional effects due to the periodic rotation of birefringence axes are included by constructing the exact two-soliton solution of the system (1). In particular, we point out that interaction can be used as a switch to suppress or to induce soliton switching, if we inject a multisoliton as an input pulse in the periodically twisted birefringent fiber.

The coupled system (1) reduces to the integrable Manakov model [5],

$$iq_{1Mz} + q_{1Mtt} + 2\mu(|q_{1M}|^2 + |q_{2M}|^2)q_{1M} = 0, \quad (2)$$

$$iq_{2Mz} + q_{2Mtt} + 2\mu(|q_{1M}|^2 + |q_{2M}|^2)q_{2M} = 0$$

(the subscript  $M$  refers to the Manakov model) under the transformation [4]

$$\begin{aligned} q_1 &= \cos(\theta/2)e^{i\Gamma z}q_{1M} - \sin(\theta/2)e^{-i\Gamma z}q_{2M}, \\ q_2 &= \sin(\theta/2)e^{i\Gamma z}q_{1M} + \cos(\theta/2)e^{-i\Gamma z}q_{2M}, \end{aligned} \quad (3)$$

where  $\Gamma = (\rho^2 + \kappa^2)^{1/2}$  and  $\theta = \tan^{-1}(\kappa/\rho)$ . Bélanger and Paré [10] and also briefly Agrawal [1] have shown that the system (1) without linear self-coupling ( $\rho=0$ ) has simple solitary wave solutions exhibiting energy exchange between the modes. By using the one-soliton solution of the Manakov model (2) in Eq. (3), Potasek [4] has pointed out the possibility of periodic intensity exchange between the orthogonally polarized modes  $q_1$  and  $q_2$  in the coupled system (1), when both the linear coupling constants  $\kappa$  and  $\rho$  are present. An interesting question arises here when one considers multisoliton solutions for the Manakov model that admits both elastic and inelastic (shape changing) types of collisions depending upon the initial conditions or arbitrary parameters as shown in Ref. [9]. Then how do the switching and energy sharing properties get modified for the multisoliton solutions of the system (1)? We show in this paper that indeed novel features in the intensity sharing and different switching properties do arise when the most general two-soliton solution is considered for Eq. (1).

For our analysis we make use of the general two-soliton solution of the Manakov system (2), reported in [9], and obtain the corresponding two-soliton solution of Eq. (1) through Eq. (3). It has the form

$$\begin{aligned} q_1 &= \{ [\cos(\theta/2)e^{i\Gamma z}\alpha_1 - \sin(\theta/2)e^{-i\Gamma z}\beta_1]e^{\eta_1} \\ &\quad + [\cos(\theta/2)e^{i\Gamma z}\alpha_2 - \sin(\theta/2)e^{-i\Gamma z}\beta_2]e^{\eta_2} \\ &\quad + [\cos(\theta/2)e^{i\Gamma z + \delta_1} - \sin(\theta/2)e^{-i\Gamma z + \delta'_1}]e^{\eta_1 + \eta_1^* + \eta_2} \\ &\quad + [\cos(\theta/2)e^{i\Gamma z + \delta_2} \\ &\quad - \sin(\theta/2)e^{-i\Gamma z + \delta'_2}]e^{\eta_1 + \eta_2 + \eta_2^*} \} / D_n, \\ q_2 &= \{ [\sin(\theta/2)e^{i\Gamma z}\alpha_1 + \cos(\theta/2)e^{-i\Gamma z}\beta_1]e^{\eta_1} \\ &\quad + [\sin(\theta/2)e^{i\Gamma z}\alpha_2 + \cos(\theta/2)e^{-i\Gamma z}\beta_2] \\ &\quad + [\sin(\theta/2)e^{-i\Gamma z + \delta_1} + \cos(\theta/2)e^{-i\Gamma z + \delta'_1}]e^{\eta_1 + \eta_2^* + \eta_2} \\ &\quad + [\sin(\theta/2)e^{i\Gamma z + \delta_2} + \cos(\theta/2)e^{-i\Gamma z + \delta'_1}] \\ &\quad \times e^{\eta_1 + \eta_2 + \eta_2^*} \} / D_n \end{aligned} \quad (4)$$

(the asterisk denotes complex conjugation). Here

$$\begin{aligned} D_n &= 1 + \exp(\eta_1 + \eta_1^* + R_1) + \exp(\eta_1 + \eta_2^* + \delta_0) \\ &\quad + \exp(\eta_1^* + \eta_2 + \delta_0^*) + \exp(\eta_2 + \eta_2^* + R_2) \\ &\quad + \exp(\eta_1 + \eta_1^* + \eta_2 + \eta_2^* + R_3) \end{aligned}$$

and

$$\eta_j = k_j(t + ik_j z), \quad j=1,2.$$

The parameters

$$\begin{aligned} \exp(\delta_1) &= (k_1 - k_2)(\alpha_1\kappa_{21} - \alpha_2\kappa_{11}) / (k_1 + k_1^*)(k_1^* + k_2), \\ \exp(\delta_2) &= (k_2 - k_1)(\alpha_2\kappa_{12} - \alpha_1\kappa_{22}) / (k_2 + k_2^*)(k_1 + k_2^*), \\ \exp(\delta'_1) &= (k_1 - k_2)(\beta_1\kappa_{21} - \beta_2\kappa_{11}) / (k_1 + k_1^*)(k_1^* + k_2), \\ \exp(\delta'_2) &= (k_2 - k_1)(\beta_2\kappa_{12} - \beta_1\kappa_{22}) / (k_2 + k_2^*)(k_1 + k_2^*), \\ \exp(\delta_0) &= \kappa_{12} / (k_1 + k_2^*), \exp(R_1) = \kappa_{11} / (k_1 + k_1^*), \\ \exp(R_2) &= \kappa_{22} / (k_2 + k_2^*), \\ \exp(R_3) &= |k_1 - k_2|^2 (\kappa_{11}\kappa_{22} - \kappa_{12}\kappa_{21}) / (k_1 + k_1^*) \\ &\quad \times (k_2 + k_2^*) |k_1 + k_2^*|^2, \end{aligned}$$

and

$$\kappa_{ij} = \mu(\alpha_i\alpha_j^* + \beta_i\beta_j^*)(k_i + k_j^*)^{-1}, \quad i, j=1,2.$$

The six arbitrary complex parameters  $\alpha_1$ ,  $\alpha_2$ ,  $\beta_1$ ,  $\beta_2$ ,  $k_1$ , and  $k_2$  determine the amplitude, velocity, and phase of the asymptotic soliton forms of Eq. (4). Now in order to bring out the nature of the solitons of Eq. (1) and their interactions including exchange of energy we carry out an asymptotic analysis of the solution (4). To be specific we choose the arbitrary complex parameters  $k_i, i=1,2$ , as  $k_{1I} > k_{2I}$ ,  $k_{1R} > 0$  and  $k_{2R} > 0$  (here subscripts  $I$  and  $R$  refer to the imaginary and real parts).

(I) *Limit  $z \rightarrow -\infty$* : As  $z \rightarrow -\infty$  we can identify two independent solitons denoted by soliton 1 and soliton 2 with the above choices of  $k_1$  and  $k_2$ . Soliton 1 will be centered around  $\eta_{1R} = k_{1R}(t - 2k_{1I}z) \approx 0$  (when  $\eta_{2R} \rightarrow -\infty$ ) and soliton 2 will be centered around  $\eta_{2R} = k_{2R}(t - 2k_{2I}z) \approx 0$  (when  $\eta_{1R} \rightarrow \infty$ ).

(a) *Soliton 1* ( $\eta_{1R} \approx 0, \eta_{2R} \rightarrow -\infty$ ):

$$\begin{aligned} q_1 &\cong [\cos(\theta/2)e^{i\Gamma z}A_{1M}^{1-} - \sin(\theta/2)e^{-i\Gamma z}A_{2M}^{1-}]q^{1-}, \\ q_2 &\cong [\sin(\theta/2)e^{i\Gamma z}A_{1M}^{1-} + \cos(\theta/2)e^{-i\Gamma z}A_{2M}^{1-}]q^{1-}, \end{aligned} \quad (5)$$

where  $q^{1-} = k_{1R} \exp(i\eta_1) \text{sech}(\eta_{1R} + R_1/2)$ ,  $\eta_{1I} = k_{1I}t + (k_{1R}^2 - k_{1I}^2)z$ ,  $(A_{1M}^{1-}, A_{2M}^{1-}) = [\mu(\alpha_1\alpha_1^* + \beta_1\beta_1^*)]^{-1/2}(\alpha_1, \beta_1)$ , and  $|A_{1M}^{1-}|^2 + |A_{2M}^{1-}|^2 = 1/\mu$ . Here  $\mu^{+1/2}(A_{1M}^{1-}, A_{2M}^{1-})$  refers to the polarization unit vector of the Manakov one-soliton solution, which can be obtained from Eq. (5) by substituting  $\theta = \Gamma = 0$ , superscripts  $1-$  denote soliton 1 at the limit  $z \rightarrow -\infty$ , and subscripts 1 and 2 refer to the modes  $q_1$  and  $q_2$ . Equation (5) exhibits the same form of the one-soliton solution of Eq. (1) reported by Potasek in Ref. [4]. If we parametrize [8] the unit polarization vector as  $\mu^{+1/2}(A_{1M}^{1-}, A_{2M}^{1-}) = (\cos(\theta_p^{1-}) \exp(i\alpha_p^{1-}), \sin(\theta_p^{1-}) \exp(i\alpha_p^{1-}))$  then we can identify  $\theta_p^{1-}$  as the polarization angle and the phases  $\alpha_{p1}^{1-} \neq \alpha_{p2}^{1-}$  correspond to the state of elliptical polarization.

(b) *Soliton 2* ( $\eta_{2R} \approx 0, \eta_{1R} \rightarrow \infty$ ):

$$\begin{aligned} q_1 &\cong [\cos(\theta/2)e^{i\Gamma z}A_{1M}^{2-} - \sin(\theta/2)e^{-i\Gamma z}A_{2M}^{2-}]q^{2-}, \\ q_2 &\cong [\sin(\theta/2)e^{i\Gamma z}A_{1M}^{2-} + \cos(\theta/2)e^{-i\Gamma z}A_{2M}^{2-}]q^{2-}, \end{aligned} \quad (6)$$

where

$$q^{2-} = k_{2R} \exp(i\eta_{2l}) \operatorname{sech}[\eta_{2R} + (R_3 - R_1)/2],$$

$$\eta_{2l} = k_{2l}t + (k_{2R}^2 - k_{2l}^2)z,$$

$$(A_{1M}^{2-}, A_{2M}^{2-}) = (a_1/a_1^*)c[\mu(\alpha_2\alpha_2^* + \beta_2\beta_2^*)]^{-1/2} \\ \times [(\alpha_1\beta_1)\kappa_{11}^{-1} - (\alpha_2\beta_2)\kappa_{21}^{-1}], \\ |A_{1M}^{2-}|^2 + |A_{2M}^{2-}|^2 = 1/\mu,$$

in which

$$a_1 = (k_1 + k_2^*)(k_1 - k_2)(\alpha_1^*\alpha_2 + \beta_1^*\beta_2)]^{1/2}$$

and

$$c = [1/|\kappa_{12}|^2 - 1/\kappa_{11}\kappa_{22}]^{1/2}.$$

It is interesting to note from Eqs. (5) and (6) that the form of  $A_{1M}^{2-}$  and  $A_{2M}^{2-}$  in Eq. (6) differs from the values  $A_{1M}^{1-}$  and  $A_{2M}^{1-}$  in Eq. (5) and the former contains more parameters, even though Eq. (6) is an exact one-soliton solution of the system (1) just like the solution (5). Further, in the special case  $\alpha_1:\alpha_2 = \beta_1:\beta_2$ , the form of Eq. (6) reduces to the form of Eq. (5) with parameters specifying soliton 2. So Eq. (6) may be considered as the most general one-soliton solution form of Eq. (1).

*II. Limit  $z \rightarrow \infty$ :* We now analyze the form of the solitons after interactions as  $z \rightarrow \infty$ .

(a) *Soliton 1* ( $\eta_{1R} \approx 0, \eta_{2R} \rightarrow \infty$ ):

$$q_1 \cong [\cos(\theta/2)e^{i\Gamma z}A_{1M}^{1+} - \sin(\theta/2)e^{-i\Gamma z}A_{2M}^{1+}]q^{1+}, \\ q_2 \cong [\sin(\theta/2)e^{i\Gamma z}A_{1M}^{1+} + \cos(\theta/2)e^{-i\Gamma z}A_{2M}^{1+}]q^{1+}, \quad (7)$$

where

$$q^{1+} = k_{1R} \exp(i\eta_{1l}) \operatorname{sech}[\eta_{1R} + (R_3 - R_2)/2], \\ (A_{1M}^{1+}, A_{2M}^{1+}) = (a_2/a_2^*)c[\mu(\alpha_1\alpha_1^* + \beta_1\beta_1^*)]^{-1/2} \\ \times [(\alpha_1\beta_1)\kappa_{12}^{-1} - (\alpha_2\beta_2)\kappa_{22}^{-1}], \\ |A_{1M}^{1+}|^2 + |A_{2M}^{1+}|^2 = 1/\mu,$$

in which

$$a_2 = (k_2 + k_1^*)(k_1 - k_2)(\alpha_1\alpha_2^* + \beta_1\beta_2^*)^{1/2}.$$

Note that  $A_{1M}^{1+} \neq A_{1M}^{1-}$  and  $A_{2M}^{1+} \neq A_{2M}^{1-}$ , except when  $\alpha_1:\alpha_2 = \beta_1:\beta_2$ , corresponding to pure elastic collision in the Manakov model [9].

(b) *Soliton 2* ( $\eta_{2R} \approx 0, \eta_{1R} \rightarrow -\infty$ ):

$$q_1 \cong [\cos(\theta/2)e^{i\Gamma z}A_{1M}^{2+} - \sin(\theta/2)e^{-i\Gamma z}A_{2M}^{2+}]q^{2+}, \\ q_2 \cong [\sin(\theta/2)e^{i\Gamma z}A_{1M}^{2+} + \cos(\theta/2)e^{-i\Gamma z}A_{2M}^{2+}]q^{2+}, \quad (8)$$

where

$$q^{2+} = k_{2R} \exp(i\eta_{2l}) \operatorname{sech}(\eta_{2R} + R_2/2), \\ (A_{1M}^{2+}, A_{2M}^{2+}) = [\mu(\alpha_2\alpha_2^* + \beta_2\beta_2^*)]^{-1/2}(\alpha_2\beta_2),$$

and

$$|A_{1M}^{2+}|^2 + |A_{2M}^{2+}|^2 = 1/\mu.$$

Here also  $A_{1M}^{2+} \neq A_{1M}^{2-}$  and  $A_{2M}^{2+} \neq A_{2M}^{2-}$ , unless  $\alpha_1:\alpha_2 = \beta_1:\beta_2$ .

Now we recognize from Eqs. (5)–(8) that not only the phase factors but also the overall shapes of the solitons get modified (due to the intensity redistribution among the solitons) after undergoing interaction when the two coupled one solitons (5) and (6) move from  $z \rightarrow -\infty$  to  $z \rightarrow \infty$  as shown in Eq. (7) and Eq. (8). To facilitate the understanding of the above behavior with reference to the optical soliton switching between the orthogonally polarized modes, it is convenient to obtain the oscillating parts of the intensities associated with the asymptotic forms (5)–(8) as

$$\left| \frac{q_l(z,t)}{q^{n\mp}(z,t)} \right|^2 = |A_{lM}^{n\mp}|^2 \cos^2(\theta/2) + |A_{jM}^{n\mp}|^2 \sin^2(\theta/2) \\ + (-1)^l |A_{lM}^{n\mp}| |A_{jM}^{n\mp}| \sin(\theta) \\ \times \cos(2\Gamma z + \phi^{n\mp}), \\ l, j = 1, 2 (l \neq j), \quad z \rightarrow \mp \infty \quad (9)$$

where  $\phi^{n\mp} = \tan^{-1}(A_{1M}^{n\mp}/A_{1MR}^{n\mp}) - \tan^{-1}(A_{2M}^{n\mp}/A_{2MR}^{n\mp})$ . The presence or absence of the last term involving the factor  $\cos(2\Gamma z + \phi^{n\mp})$  plays a crucial role in the switching behavior of solitons as demonstrated below.

As we have mentioned before, the values of  $A_{jM}^{n\mp}$  ( $j, n = 1, 2$ ) change to new values  $A_{jM}^{n\pm}$  due to the collision between two copropagating solitons, namely, soliton 1 and soliton 2, without violating the condition  $|A_{1M}^{n\mp}|^2 + |A_{2M}^{n\mp}|^2 = 1/\mu$ . The amount of change ( $A_{jM}^{n\pm} - A_{jM}^{n\mp}$ ) can be estimated by assigning suitable values to the arbitrary parameters  $k_1, k_2, \alpha_1, \alpha_2, \beta_1,$  and  $\beta_2$ , appearing in the expressions for  $A_{jM}^{n\mp}$  ( $j, n = 1, 2$ ). Further, from Eq. (9) it is obvious that depending upon the values of  $|A_{jM}^{n\mp}|$ , the nature of switching dynamics supported by the system (1) also changes. The change in  $A_{jM}^{n\mp}$  can be parametrized as  $(\cos(\theta_p^{n\mp})\exp(i\alpha_{p1}^{n\mp}), \sin(\theta_p^{n\mp})\exp(i\alpha_{p2}^{n\mp}))$ , where as long as the phases  $\alpha_{p1}^{n\mp} \neq \alpha_{p2}^{n\mp}$  the state of polarization is preserved during interaction. Therefore, in general without affecting the state of polarization, the switching dynamics can be changed just by changing  $A_{jM}^{n\mp}$  with the help of the polarization angle. In the following we briefly discuss the different changes which can occur in the intensity exchange between  $q_1$  and  $q_2$  modes with respect to soliton 1 and soliton 2 due to the above-mentioned collision by considering  $|q_l/q^{n-}|^2$  and  $|q_l/q^{n+}|^2$  defined in Eq. (9) for each  $l = 1, 2$  and  $n = 1, 2$  value.

*Case 1:* All the  $|A_{jM}^{n\mp}|$ 's ( $j, n = 1, 2$ ) are nonzero. In this case due to the presence of the  $\cos(2\Gamma z + \phi^{n\mp})$  term on the right hand side of Eq. (9), there is a periodic intensity switching which is always present in both the solitons and in both the components before as well as after the interaction. Of course the conservation relations  $|A_{1M}^{n-}|^2 + |A_{2M}^{n-}|^2 = |A_{1M}^{n+}|^2 + |A_{2M}^{n+}|^2 = 1/\mu$  for the total intensity are always valid. However, the switching dynamics appearing before and after interaction are not similar in form, due to the condition  $A_{jM}^{n\pm} \neq A_{jM}^{n-}$ ,  $j, n = 1, 2$ , except when  $\alpha_1:\alpha_2 = \beta_1:\beta_2$  as mentioned

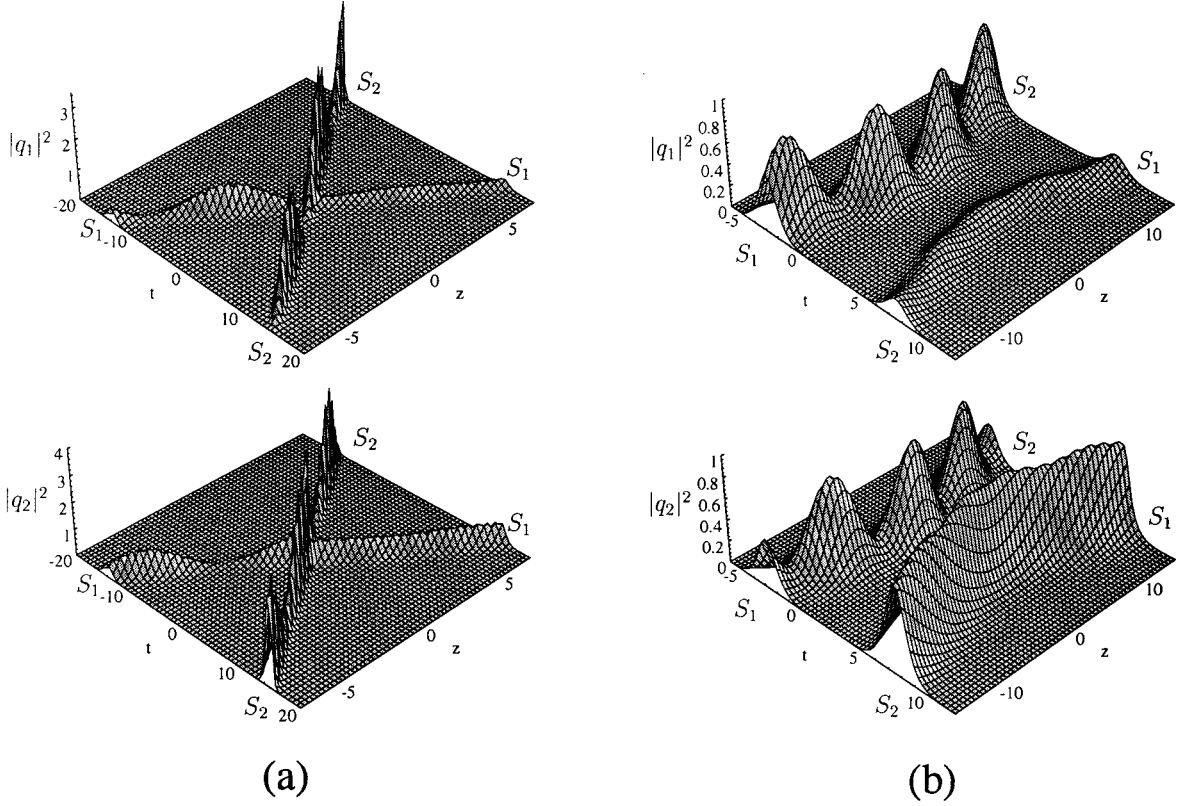


FIG. 1. Typical evolution of the intensity profiles  $|q_1|^2$  and  $|q_2|^2$  of the two-soliton solution (4), (a) showing the suppression in switching between the two modes of  $s_1$  soliton and (b) showing the suppression in switching of  $s_1$  soliton and enhancement in  $s_2$  soliton (while undergoing a large phase shift) for the parameter values given in the text.

before, giving rise to a partial suppression or enhancement of the periodically varying intensities.

*Case 2:* Any one of the  $|A_{jM}^{n\pm}|$ 's is zero and others are nonzero. For example, if  $|A_{1M}^{1+}| \sim 0$ , corresponding to the condition  $\alpha_2(k_2+k_2^*)(\alpha_1\alpha_2^*+\beta_1\beta_2^*)=\alpha_1(|\alpha_2|^2+|\beta_2|^2)(k_1+k_2^*)$ , then the switching in the intensity of soliton 1 gets fully suppressed in both the modes  $q_1$  and  $q_2$ , while it persists for the other soliton. This is illustrated in Fig. 1(a) for the chosen parameter values, namely,  $k_1=1+i$ ,  $k_2=2-i$ ,  $\alpha_1=\beta_1=\beta_2=1$ ,  $\alpha_2=(39+i80)/89[\approx \exp(i64^\circ)]$ ,  $\rho=0.25$ , and  $\varkappa=0.5$ , for which  $|A_{1M}^{1-}| \sim 0.7$ ,  $|A_{1M}^{2-}| \sim 0.5$ ,  $|A_{2M}^{1-}| \sim 0.7$ ,  $|A_{2M}^{2-}| \sim 0.86$ ,  $|A_{1M}^{1+}| \sim 0.06$ ,  $|A_{1M}^{2+}| \sim 0.7$ ,  $|A_{2M}^{1+}| \sim 0.99$ , and  $|A_{2M}^{2+}| \sim 0.7$ , satisfying the condition that  $|A_{1M}^{1+}| \sim 0$ . A similar phenomenon can be seen if  $|A_{2M}^{1+}| \sim 0$  instead of  $|A_{1M}^{1+}| \sim 0$ . But if we choose  $|A_{jM}^{2+}| \sim 0$  ( $j=1$  or  $j=2$ ) the periodic intensity exchange with respect to soliton 2 will be suppressed, while it persists in soliton 1. On the other hand, if  $|A_{jM}^{n-}| \sim 0$  ( $j=1$  or  $j=2$ ;  $n=1$  or  $n=2$ ), then there is no switching in the intensity of soliton  $n$  before interaction, but the switching appears after interaction in that soliton and so there is an inducement of switching due to the interaction. Thus the interaction itself acts as a switch to suppress or to enhance the switching dynamics.

*Case 3:* Any two of the  $|A_{jM}^{n\pm}|$ 's are zero and the others are nonzero without violating the conservation conditions. For concreteness, let  $|A_{1M}^{n+}| \sim 0$  ( $n=1,2$ ), which implies the condition  $\alpha_1 \sim \alpha_2 \sim 0$ . It implies that  $|A_{1M}^{n-}|$  ( $n=1,2$ ) should also simultaneously vanish. Consequently there is no switching between the modes  $q_1$  and  $q_2$  either before or after in-

teraction and there will be only inelastic (shape changing) scattering as discussed for the Manakov model in Ref. [9]. Similar observations can also be made if one makes any two of the  $|A_{jM}^{n\pm}|$ 's vanish. However, one can identify the interesting possibility of switching existing in soliton 1 only before interaction, which gets interchanged with soliton 2 after interaction, by allowing one of the two  $|A_{jM}^{n\pm}|$ 's and another one of the  $|A_{jM}^{n\pm}|$ 's simultaneously to take the value zero, corresponding to the condition

$$\begin{aligned} & [ |k_1+k_2^*|^2 (|\alpha_1|^2+|\beta_1|^2) (|\alpha_2|^2+|\beta_2|^2) ] \\ & = [(k_1+k_1^*)(k_2+k_2^*) |\alpha_1\alpha_2^*+\beta_1\beta_2^*|^2]. \end{aligned}$$

Figure 1(b), for the chosen parameters, namely,  $k_1=1+i0.1$ ,  $k_2=1-i0.1$ ,  $\alpha_1=0.86+i0.5$ ,  $\alpha_2=0.5+i0.86$ ,  $\beta_1=0.7+i0.72$ ,  $\beta_2=0.44+i0.9$ , and  $\rho=\varkappa=0.25$ , for which  $|A_{1M}^{1-}| \sim 0.7$ ,  $|A_{2M}^{2-}| \sim 0.05$ ,  $|A_{1M}^{1+}| \sim 0.7$ ,  $|A_{2M}^{2-}| \sim 0.99$ ,  $|A_{1M}^{1+}| \sim 0.04$ ,  $|A_{2M}^{1+}| \sim 0.7$ ,  $|A_{2M}^{1+}| \sim 0.99$ , and  $|A_{2M}^{2+}| \sim 0.7$  satisfying the above condition, shows that the interaction induces the periodic intensity exchange between the two modes of soliton 2 while it suppresses the switching dynamics in soliton 1.

To conclude, by studying the interaction between two co-propagating solitons in the periodically twisted birefringent fiber with reference to soliton switching, we have observed several possible ways to use interaction as a switch to suppress or to induce the switching dynamics. The basic underlying mechanism of such possibilities is the inelastic (shape changing) nature of the soliton interaction which arises es-



essentially due to changes in the polarization angle and so in the overall amplitude of the solitons. Since Mollenauer *et al.* [8] have demonstrated polarization scattering by soliton-soliton collision, it should also be possible to experimentally study the phenomena described in this paper by using specially fabricated optical fibers. These possibilities should

have important ramifications in nonlinear switching devices like the rocking rotator.

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